HW 8

Problem 1 (8.51) The double exponential distribution is

$$f(x;\theta) = \frac{1}{2}\exp^{-|x-\theta|}\mathbb{I}\{-\infty < x < \infty\}.$$

For an IID sample of size $n = 2m+1$, show that the MLE of θ is the median of the sample, i.e. the observation such that half of the rest of the observations are smaller and half are larger. (Hint: Since $g(x) =\mid x \mid$ is not differentiable, draw a picture for a small value of n to figure out what's going on with the likelihood.)

Problem 2 (8.60) Let $X_1, \ldots, X_n$ be IID from an exponential distribution with density

$$f(x;\theta) = \frac{1}{\tau} \exp^{-x/\tau} \mathbb{I}\{0 \leq x < \infty\}.$$

(a) Find the MLE of τ and find its (exact) sampling distribution.

(b) Find the form of an exact CI for τ.

(c) Find the sampling expectation and variance of the MLE. (Hint: The sum of the X_i follows a gamma distribution.)

Problem 3 (8.60) Let $X_1, \ldots, X_n$ be IID from an exponential distribution with density

$$f(x;\theta) = \frac{1}{\tau} \exp^{-x/\tau} \mathbb{I}\{0 \leq x < \infty\}.$$

(a) Use the CLT to find a normal approximation to the sampling distribution of the MLE for τ.

(b) Find the form of an approximate CI for τ.