HW 3

Problem 1 (4.65) A *random sum* is of the type $T = \sum_{i=1}^{N} X_i$ where N is a RV with a finite expectation and the X_i are RVs that are independent of N and have a common mean $E(X) = \mu$. Consider the random sum where N is the number of jobs in a queue for maintenance repairs to a dorm and X_i is the service time required for the ith job in the queue. Thus $T = \sum_{i=1}^{N} X_i$ is the time required to serve all the jobs in the queue. According to the Law of Total Expectation,

$$E(T) = E[E(T \mid N)].$$

Since $E(T \mid N = n) = nE(X)$, $E(T \mid N) = NE(X)$ and thus

$$E(T) = E[NE(X)] = E(N)E(X).$$

How has the assumption that N and each X_i are independent been used here?

Problem 2 (Ch 3) Suppose $X_1, \ldots, X_n$ are IID $Poisson(\lambda)$ RVs. Find the exact distribution of $S_n = \sum_{i=1}^n X_i$.

Problem 3 (5.11) As in the previous problem, suppose $X_1, \ldots, X_n$ are IID $Poisson(\lambda)$ RVs and $S_n = \sum_{i=1}^{n} X_i$. What is the behavior of the distribution of S_n as $n \to \infty$? Does the CLT apply? Why/why not?