HW 14

Problem 1 (9.12) Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with density $f(x; \theta) = \theta \exp\{-\theta x\}$ Derive a likelihood ratio test of $H_0 : \theta = \theta_0$ vs $H_A : \theta \neq \theta_0$ and show that the rejection region is of the form $\{\bar{X} \exp\{-\theta_0 \bar{X}\} \leq c\}$.

Problem 2 (9.17) Let $X \sim N(0, \sigma^2)$ and consider testing $H_0 : \sigma = \sigma_0$ vs $H_A : \sigma = \sigma_1$ where $\sigma_1 > \sigma_0$ are fixed values.

(a) What is the likelihood ratio as a function of the data point, x?

(b) What values of x favor H_0?

(c) What is the rejection region of a level α test?

Problem 3 (9.17) Let $X_1, \ldots, X_n$ be IID RVs with distribution $N(0, \sigma^2)$. Again, consider testing $H_0 : \sigma = \sigma_0$ vs $H_A : \sigma = \sigma_1$ where $\sigma_1 > \sigma_0$ are fixed values.

(a) What is the likelihood ratio as a function of the data points, $x_1, \ldots, x_n$?

(b) What values of $x_1, \ldots, x_n$ favor H_0?

(c) What is the rejection region of a level α test?

(d) Is this test uniformly most powerful for testing $H_0 : \sigma = \sigma_0$ vs $H_A : \sigma > \sigma_0$? Why or why not?