HW 10

Problem 1 (8.57) Suppose that $X_1, \ldots, X_n$ are IID from a $N(\mu, \sigma^2)$. You may use the fact that

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{(n-1)},$$

and that the mean and variance of a Chi-square RV with r degrees of freedom are r and $2r$, respectively. (See Section 6.3 of your textbook for proof.)

(a) Which of the following estimates is unbiased?

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

(b) Which of the estimates in part (a) has a smaller MSE?

Problem 2 (8.63) Suppose 100 items are randomly sample from a manufacturing process and 3 are found to be defective. A beta prior is used for the unknown proportion of defective items, θ. Consider two cases for the prior (1) $a = b = 1$ and (2) $a = 0.5$ and $b = 5$. Plot the two different posterior distributions and find each of the posterior means. Explain the differences between these two posteriors.

Problem 3 (8.64) In the same setting as Problem 2, let $X = \begin{cases} 1, & \text{if item is defective} \\ 0, & \text{otherwise} \end{cases}$.

(a) For each choice of prior, what is the marginal distribution of X *before* the sample of 100 is drawn?

(b) For each choice of prior, what is the marginal distribution of X *after* the sample is observed? (Hint: Use the posterior distribution of θ.)