Quiz 3
Stat 61
Due to Gradescope by 12:00AM Oct 28 *

Suppose we observe $X_1, \ldots, X_n$ IID data points from an $\text{Exp}(1/\mu)$ distribution (i.e. the density of X_1 is $f(x_1;\theta) = \frac{1}{\mu}e^{-x_1/\mu}$ for $x_1 > 0$). You may use without proof that $E(X_1) = \mu$ and that

$$W = \frac{2n}{\mu}\bar{X} \sim \chi^2_{(2n)}.$$

(**Hint:** You may want to use R to help with quantile and/or probability calculations for a chi-squared RV. E.g. Type *?qchisq()* into the R command line for more info on how to use these functions.)

For Problems 1-2, suppose we observe a sample of size $n = 5$ and we are considering an $\alpha = 0.025$ level test of $H_0 : \mu = 100$.

1. Match the following alternative hypotheses to their corresponding power.

$H_1 : \mu = \mu_1$	**Power**
$\mu_1 = 1$	0.03
$\mu_1 = 20$	0.91
$\mu_1 = 99$	1

2. What is the test statistic, $T(X_1, \ldots, X_n)$, and the rejection region, $A_{\alpha=0.025}$ for the uniformly most powerful test of $H_0 : \mu = 100$ vs $H_1 : \mu > 100$?

$T(X_1, \ldots, X_n) =$ ________________

$A_{\alpha=0.025} =$ ______________________________

3. For an arbitrary value of $\bar{X}$, give the expression for a 2-sided 95% confidence interval for μ. (You can earn **partial** credit by only finding a CI for $1/\mu$ instead.)

$CI =$ ______________________________

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