Quiz 1 - Solutions
Stat 61
Due to Gradescope by 12:00AM Sept 17

For problems 1-2 consider $X_1, \dots, X_n$, an IID sample from a Geometric distribution with rate θ, i.e. $P(X_i = x_i) = \theta(1-\theta)^{x_i - 1}$ for $x_i = 1, 2, 3, \dots$. You are asked to compare several different estimators for θ. For Bayesian estimation, you may use without proof the fact that the prior distribution $\theta \sim Beta(a, b)$ produces a posterior that also follows a Beta distribution.

1. Match the following estimators on the right to the name of the approach that produced it on the left. (Note: All methods must be matched to an estimator but do not assume a one-to-one nor onto mapping from the procedures to the estimates.)

Procedure
Method of moments
Maximum likelihood
(Bayesian) Posterior mean with $Beta(1, 2)$ prior

Estimate
$\hat{\theta}_1 = \begin{cases} 1, & \text{if } x_1 = 1 \\ 0, & \text{if } x_1 \geq 2 \end{cases}$
$\hat{\theta}_2 = \dfrac{1+n}{3+n\bar{x}}$
$\hat{\theta}_3 = \dfrac{1}{\bar{x}}$

2. Circle the estimator(s) below that is/are unbiased for θ? (Hint: How could you use Jensen's inequality to prove that an estimator is biased?)

$\hat{\theta}_1$ $\hat{\theta}_3$

3. Circle the estimator(s) below that is/are consistent for θ?

$\hat{\theta}_1$ $\hat{\theta}_3$

Submitting instructions
Upload a scanned, completed version of this page to Gradescope by the deadline. You should also upload additional pages that show your work as scanned PDFs. Any additional pages must be clearly labeled and display how you arrived at the answers on this page. You will not receive full credit for handing in solutions without any work or justification.