Different forms of One-Way ANOVA Models

The three equations below are all valid forms of a one-way ANOVA model. Here, the index $j = 1, \ldots, k$ represents the group (or treatment level) of the single categorical predictor variable that has k different levels.

Group effects form

In this form of the ANOVA model, we can compare the effect of each of the groups/levels on Y, relative to the overall behavior of Y summarized in the grand mean of Y, μ.

$$Y = \mu + \alpha_j + \epsilon$$

Group means form

In this equivalent form of the ANOVA model, the group effects are hidden inside the means of Y for each level.

$$Y = \mu_j + \epsilon, \text{ where } \mu_j = \mu + \alpha_j$$

MLR form

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_{k-1} x_{k-1} + \epsilon, \text{ where } x_j = \begin{cases} 1, & \text{if observation is in group } j \\ 0, & \text{otherwise} \end{cases}$$

The tricky thing about the MLR form is that it is not so easy to read off the different group effects and/or group means from the model output. If we consider each of the coefficients in terms of the notation above we have that

$$\beta_0 = \mu_k = (\mu + \alpha_k)$$

which is easy enough to interpret, but

$$\beta_1 = \mu_k - \mu_1 = (\mu + \alpha_k) - (\mu + \alpha_1) = \alpha_k - \alpha_1$$

and so on for all other coefficients,

$$\beta_{k-1} = \mu_k - \mu_{k-1} = (\mu + \alpha_k) - (\mu + \alpha_{k-1}) = \alpha_k - \alpha_{k-1}.$$

This means that based on the MLR form of the ANOVA model, if we wanted to find the group mean for group $k - 1$ we would need to compute

$$\mu_{k-1} = \mu_k - \beta_{k-1} = \beta_0 - \beta_{k-1},$$

and, if we want to find the group effect for any of the $1, \ldots, k$ groups, we need to first find μ, the overall mean of Y.